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تحلیل مکانی بارش: مقایسه روش‌های کریجینگ با روش‌های متداول

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چکیده

یکی از مسائل مهم در مدیریت منابع آب، برآورد اطلاعات در مناطقی است که داده‌های آن اندازه‌گیری نشده و یا دارای داده‌های مفقوده می‌باشد. تخمین داده‌های نامعلوم در نقاط مختلف با استفاده از روش‌های درون‌یابی بر روی داده‌های مشاهداتی در محدوده مورد مطالعه صورت می‌گیرد. برای محاسبه و تحلیل مکانی داده‌های هیدرولوژی مانند بارش، روش‌های متعدد درون‌یابی از روش‌های ساده خطی تا روش‌های پیچیده چند متغیره وجود دارد. دقت نتایج برای تخمین داده‌ها در هریک از روش‌ها با توجه به قیود و داده‌های لازم متفاوت می‌باشد، به همین منظور روش‌های متفاوتی برای تحلیل داده‌های مکانی و زمانی به‌کار گرفته می‌شود. در این مقاله از روش‌های متداول درون‌یابی و زمین آماری مانند کریجینگ در تحلیل مکانی و تخمین متوسط بارش منطقه‌ای ماهانه بر روی ۳۸ ایستگاه بارش برای دوره زمانی ۱۹۶۷ تا ۲۰۰۵ در محدوده غرب کشور استفاده شده است. همچنین نتایج به‌دست آمده در بسیاری از ایستگاه‌ها با نتایج ثبت شده در هر ایستگاه مقایسه گردید. نتایج نشان می‌دهد که روش‌های متفاوت درون‌یابی می‌بایست با در نظر گرفتن شاخص‌های محلی که بر روی متغیرهای هواشناسی موثرند، استفاده گردند. با توجه به نتایج به دست آمده در محدوده مورد مطالعه، روش کریجینگ یونیورسال بهترین نتیجه را نسبت به سایر روش‌های مختلف درون‌یابی دارا می‌باشد. بر اساس بهترین نتایج به‌دست آمده از تخمین بارش منطقه‌ای با استفاده از روش‌های کریجینگ و متداول، ایستگاه‌های جدیدی در محدوده مورد مطالعه به‌منظور بهینه‌سازی ایستگاه‌های بارش پیشنهاد گردید.

کلمات کلیدی: بارش، کریجینگ، تحلیل مکانی، روش‌های متداول، بهینه‌سازی ایستگاه بارش، درون‌یابی

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Analysis of Spatial Variation of Precipitation: Comparison of Conventional and Kriging Methods

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Abstract

One of the major challenges in water resources management is estimation of missing or un-gauged data. Estimates of the unknown data in different points can be made by spatial interpolation of the observed data recorded in some special points of the study region. In case of spatial variations of precipitation data, there are many interpolation techniques changing from simple linear methods to complicated multivariate methods. Each method has its own necessities and constraints which result in different levels of accuracy and precision in the estimated values. Accordingly regarding the needed accuracy in different applications and the available time and data, different spatial analysis methods are utilized. In this paper different conventional and modern methods such as interpolation and Kriging are used for spatial analysis of the precipitation in the western part of Iran. Many different techniques have been applied to measure annual and monthly precipitation data at 38 stations for the period of 1967 to 2005 in the study area. The results are compared by estimation of the data for some stations in the region. The results showed that different methods of interpolation should be used with care considering some local characteristics that affect the climatic variables. In this case study, universal Kriging had the best performance among different interpolation techniques. Based on the best results of conventional and Kriging methods in this study, the locations of some new stations are proposed.

Keywords: Precipitation, Kriging, Spatial Analysis, Conventional Methods, Optimal Placement of Station, Interpolation

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1-Introduction

Estimates of spatial variability of precipitation are important for implementing water resources management strategies especially when dealing with flood and drought events. Climate data are only recorded at some special points at the meteorological stations. The values of climate variables at any other points in the study area must be inferred from the neighboring stations using consistent methods applicable to the study area. In the absence of proper models the accurate spatial analysis of precipitation requires a dense network of measuring gauges, which entails large installation and operating costs. A number of methods have been proposed for interpolation of precipitation data ranging from the conventional approaches such as the nearest point, moving average, and moving surface to the new approaches such as universal Kriging, anisotropic Kriging and ordinary Kriging methods.

Many of the available studies model the spatial distribution of a climate variable using interpolation methods (Kurtzman and Kadmon, 1999; Oliver and Webster, 1990; Mitas and Mitasova, 1988; Philip and Watson, 1982). The accuracy of the results of these methods is dependent on the geographic situation of the sampling points, topological relationships between different points of the study region, and the value of the measured variable. The interpolation methods consider spatial relationships among sampling points. The precipitation scheme generally varies with elevation (Spreen, 1947; Smith, 1979), and so many authors have incorporated elevation into the interpolation approaches (Martı´nez-Cob, 1996; Prudhomme and Duncan, 1999; Goovaerts, 2000).

Geostatistics, which is based on the theory of regionalized variables (Journel and Huijbregts, 1978; Goovaerts, 2000), is increasingly applied for regionalizing data because it uses the spatial correlation among adjacent observations to predict attribute values at un-gauged locations. Several investigators such as Tabios and Salas (1985) and Phillips et al. (1992) have shown that the geostatistical prediction techniques such as Kriging provide better estimates of precipitation than conventional methods. Garen et al. (1994) found that the results depend on the sampling density. They also reported that for high-resolution networks the geostatistical methods such as Kriging shows significantly greater predictive skill than simpler techniques such as the inverse square distance method. In fact, a major advantage of Kriging over conventional methods is providing a measure of prediction error (Kriging variance). Considering the elevation variation through 3 dimensional Kriging improves the interpolation results. For example, Hevesi et al. (1992) reported a significant 75% correlation between average annual precipitation and elevation recorded at 62 stations in Nevada and southeastern California.

In this study, some conventional and modern techniques were applied and compared for the regional precipitation estimation in the western part of Iran. The monthly precipitation data measured at 38 stations in the study area for the period of 1967 to 2005 were used for this purpose. Different methods were compared based on the estimated values in the stations in the study area. The histogram of the spatial analysis is estimated to evaluate the stability of applied methods. The method of spatial analysis of precipitation using different interpolation methods was described in the following section. The characteristics of the study area are introduced in the next section which is followed by discussion on the result of different methods. Finally, a summary of the results and the conclusion is given.

2- Spatial Analysis Methods

There are different interpolation techniques ranging from very simple, such as the nearest point method, to very complicated methods such as the universal kriging for spatial analysis of data. The numerical accuracy and the data needed for each of these techniques are very different. Therefore, before using any interpolation method, the users should answer questions like:

Q: Do I really need this interpolation method?

Examples of situations where Kriging could be very helpful are the mining industry, environmental research where decisions could have major economical and juridical consequences (e.g. is the area under study polluted or not) and so on.

Q: Is this method the most appropriate interpolation method for my sample set?

Before using an interpolation technique, first the assumptions of the method(s) should be considered carefully. For example, for modeling it is better to choose a straightforward interpolation method and to calculate estimated errors. It is preferable to use the Kriging methods.

Based on the answers to such questions, different conventional or modern methods may be selected to be more in agreement with the situation of the case study. Different methods including the nearest point, moving average, moving surface, ordinary Kriging, universal Kriging, and anisotropic Kriging are applied and compared in this paper. A brief description on the structure of these methods is given in the next sub-section.

2-1- Conventional Methods

- **Nearest Point**

In the Nearest Point method (also called Nearest Neighbor or Thiessen Polygons) the value, of the nearest point is assigned to the point according to the Euclidean distance.

- **Moving Average**

The Moving Average method performs a weighted averaging on point values of a variable. The output value for a point is calculated as the sum of the products of weights and point values, divided by the sum of weights. Weight values are calculated in such a way that those points close to an output point obtain large weights and points further away obtain small weights.

- **Moving Surface**

In the Moving Surface method different point values are calculated by fitting a surface through weighted point values. Weights for all points are calculated by a user-specified weight function. Weights may, for instance, be equal to the inverse distance. The weight functions are implemented in such a way that points which are further away from an output point than the user-specified limiting distance obtain a weight of zero.

2-2- Kriging Methods

Because of the limitations of the classic interpolation methods, different methods are developed to overcome problems and increase the accuracy of the predictions. Kriging is an alternative to many other point interpolation techniques. Unlike straightforward classic methods, it is an advanced statistical method based on the theory of regionalized variables. Before using Kriging, a semivariogram model should be made, which will determine the interpolation function. Furthermore Kriging is the interpolation method which gives an interpolated map, output error map with the standard errors of the estimates and their histograms. In this manner Kriging raises the quality of the predictions.

Once users decide that Kriging is the method they want to use, they should continue with the following steps:

Step 1: Examining the input data

Step 2: Calculating experimental variograms

Step 3: Modeling variograms

Step 4: Kriging interpolation

- **Ordinary 2D Kriging**

Kriging is a geostatistic method, which estimates the unknown values using the measured values at sampled points. It also calculates the estimation errors in the form of estimation variance. The simplest type of Kriging is considered as a weighted moving average as follows:

$$Z_v^* = \sum_{i=1}^{n} \lambda_i Z_{v_i} \tag{1}$$

where,

Z_v^*: The Kriging estimation, λ_i: Weight of the quantity of the i th sample, Z_{v_i}: Quantity of the i th value and n: Number of samples.

Since the Kriging estimator is one of the best linear unbiased estimators, it should satisfy two conditions:

a) There should not be any systematic errors in its estimations and consequently the average of the estimation errors must be equal to zero,

$$E\left[Z_v - Z_v^*\right] = 0 \text{ or } \sum_{i=1}^{n} \lambda_i = 1 \tag{2}$$

b) The estimation variance should be minimized after determining of the function. Solving the equations will result in a linear system of equations including n+1 equations and n+1 unknown values which can be written in the matrix form with respect to variogram (γ) as follows:

$$AX = B \tag{3}$$

where,

$$A = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} & 1 \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} & 1 \\ 1 & 1 & \ldots & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \gamma_{1v} \\ \gamma_{2v} \\ \vdots \\ \gamma_{nv} \\ 1 \end{bmatrix} \quad X = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \\ -\mu \end{bmatrix}$$

and μ is the Lagrange multiplier.

Variogram analysis is the first step in Kriging analysis. The empirical variance or the semivariance shows the degree of dependence of the samples. The value of the semivariance of a group of points depends on the distance between them. The shorter distance often results in smaller semivariance and vice versa (Kitanidis, 1999). The graph of the semivariance as a function of the distance from a specified point, is named the empirical variogram or the semivariogram which is calculated as follows (Deutch and Journel, 1998):

$$\gamma(h) = \frac{1}{2n} \sum \left[(Z(x_i) - Z(x_i + h))^2 \right] \tag{4}$$

where,

$\gamma(h)$: Semivariogram value at a distance equal to h, $Z(x_i)$: Measured value of a variable at x_i, $Z(x_i + h)$: Measured value of a variable at $x_i + h$ and n: total number of the measured points.

- **3D Kriging**

3D Kriging is almost identical to 2D Kriging. The only difference between 2D and 3D Kriging is the distance vectors that are used for calculation of semivariograms. 2D Kriging has two components including latitude and longitude difference between two considered points, but in 3D Kriging the distance vectors have three components where the difference of elevation of two points has been added to the components of 2D Kriging.

- **Universal Kriging**

Universal Kriging is a variant of Ordinary Kriging. Universal Kriging is Kriging with a local trend or drift. This local trend or drift is a continuous and slowly varying trend surface on top of which the variation to be interpolated is superimposed. The local trend is recomputed for each output point and the operation is therefore similar to the Moving Surface operation in some aspects.

The expressions for the local trend can be incorporated into the system of simultaneous equations used to find the Kriging weights. For a system of 5 input points and a local linear trend, the set of equations are as follows:

$$\begin{bmatrix} 0\ldots & \gamma(h_{12})\ldots\gamma(h_{13})\ldots\gamma(h_{14})\ldots\gamma(h_{15})\ldots 1\ldots x_1\ldots y_1 \\ \gamma(h_{21})\ldots 0\ldots & \gamma(h_{23})\ldots\gamma(h_{24})\ldots\gamma(h_{25})\ldots 1\ldots x_2\ldots y_2 \\ \gamma(h_{31})\ldots\gamma(h_{32})\ldots 0\ldots & \gamma(h_{34})\ldots\gamma(h_{35})\ldots 1\ldots x_3\ldots y_3 \\ \gamma(h_{41})\ldots\gamma(h_{42})\ldots\gamma(h_{43})\ldots 0\ldots & \gamma(h_{45})\ldots 1\ldots x_4\ldots y_4 \\ \gamma(h_{51})\ldots\gamma(h_{52})\ldots\gamma(h_{53}) & \gamma(h_{54})\ldots 0\ldots \quad 1\ldots x_5\ldots y_5 \\ 1\ldots\ldots \quad \ldots 1\ldots\ldots \quad 1\ldots\ldots \quad 1 & 1 \quad 0\ldots 0 \quad 0 \\ x_1 \quad x_2 \quad x_3 \quad x_4 & x_5 \quad 0\ldots 0 \quad 0 \\ y_1 \quad y_2 \quad y_3 \quad y_4 & y_5 \quad 0\ldots 0 \quad 0 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ \lambda \\ \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \gamma(h_{p1}) \\ \gamma(h_{p2}) \\ \gamma(h_{p3}) \\ \gamma(h_{p4}) \\ \gamma(h_{p5}) \\ \cdots 1 \\ \ldots x_p \\ \ldots y_p \end{bmatrix} \quad (5)$$

where:

h_{ik} : is the distance between input point i and input point k,

h_{pi} : is the distance between output point p and input point i

γ (h_{ik}) :is the value of the semi-variogram model for distance h_{ik}, i.e. the semi-variogram value for the distance between input point i and input point k

γ (h_{pi}): is the value of the semi-variogram model for the distance h_{pi} , i.e. the semi-variogram value for the distance between output point p and input point i

x_i, y_i: are the XY-coordinates of input point i

w_i: is a weight factor for input point i

λ : is a Lagrange multiplier, used to minimize possible estimation error

a_1, a_2 :are the local trend coefficients of the first order trend

x_p, y_p: are the XY-coordinates of output point p

This matrix form has to be solved for each output point in the same way as described in Ordinary Kriging. Once the weights of the input point values are known it is possible to estimate or predict values for the output map and to calculate the error variance and the standard error.

- **Anisotropic Kriging**

When the variable under study is not varying in the same way in all directions, then anisotropy is present and the user must use the bidirectional method. Incorporating geometric anisotropy in the Kriging procedure is simply a matter of applying an affine transformation to the distances. An affine transformation keeps point distances in one direction unchanged and stretches distances in the direction perpendicular to it. In theory the procedure of this method is as follows:

1. The first step is a rotation of the x-axis to a position parallel to the presumed major or primary axis of anisotropy:

$$\begin{pmatrix} x^* \\ y^* \end{pmatrix} = Q \times \begin{pmatrix} x \\ y \end{pmatrix} \quad (6)$$

where,

$$Q = \begin{pmatrix} \cos\alpha\ldots\sin\alpha \\ -\sin\alpha\ldots\cos\alpha \end{pmatrix}$$

$\alpha = rotation\ angle$

2. The second step is to transform the ellipse into a circle

$$\begin{pmatrix} x^{**} \\ y^{**} \end{pmatrix} = D \times \begin{pmatrix} x^* \\ y^* \end{pmatrix} \quad (7)$$

where:

$$D = \begin{pmatrix} 1\ldots\ldots 0 \\ 0\ldots\ldots r \end{pmatrix}$$

r = anisotropy angle

3. Finally, geometric anisotropy can be described with an isotropic model according to:

$$\gamma(h) = \gamma(\|x\|) \quad (8)$$

where is $\gamma(h)$ =semi-variogram value and $\|x\|$ is the length of the separation vector.

3- A Case Study

The study area is a band in the western part of Iran covering about 250000 square kilometers of the west, northwest, and southwest basins (Figure 1). The data measured at 38 stations for the period 1967 to 2005 are used in this study. Elevation of gauges varies from 20 m above sea level in Ahvaz to 2220 m above sea level in Shahid Stations.

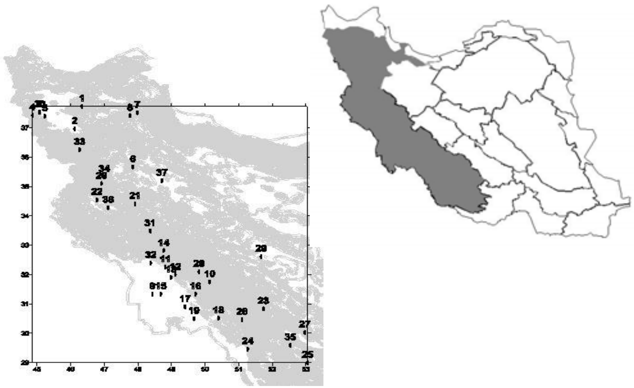


Figure 1- The Study Area and the Location of the Stations

Different conventional and modern spatial analysis methods are implemented in the study area to estimate the average precipitation of the region. GSLIB and ILWIS software have been used for this purpose. The trial and error procedure is followed in order to determine the most appropriate pixel size for spatial analysis. There are some gages in the study area which are recently established in the study area. The recorded data from these gages, in recent years are used for evaluation and comparison of different applied interpolation techniques. The steps of precipitation spatial analysis followed in this study are as follows:

- Preparing GSLIB and ILWIS software input files and analysis of variograms for selecting model parameters including, Nugget, range(h_{max}), sill, pixel size, and variogram type, only for Kriging methods (Table 1 and Figure 3).
- Spatial analysis based on the considered methods;
- Calculating average precipitation in the region;
- Developing the histograms of the interpolated precipitation data in the study area
- Estimation of rainfall in considered points of new gages
- Evaluation of estimated error of the method

4- Results

At the first step, the map of spatial distribution of the 38 considered stations in the study area has been developed and the long lead averages of monthly rainfalls have been attributed to the stations. Three conventional methods have been considered for spatial analysis of the rainfall in the study area including nearest point, moving average, and moving surface. The exponent of weighting method has been considered equal to 1 for both moving surface and moving average methods and the plane has been selected for moving surface method. The results of application of these methods including mean, median and standard deviation are presented in Table 1. The results of moving average and moving surface are more similar to each other in comparison with nearest point. There is no considerable difference in months with low rainfall but there is up to 3% difference in rainfall predictions. As in the nearest point method the amount of the nearest station is attributed to the other points and the standard deviations are high in relation to the other methods.

Five stations, established in recent years in the study area have been used as benchmarks for evaluation of the spatial analysis results by different methods. The average errors of monthly precipitation estimations at these points using different conventional interpolation techniques are presented in the last row of Table 1. The moving surface method has the least error (about 14%) and the nearest method shows the maximum error (about 21%).

In the next step the 2D and 3D empirical variagram of the precipitation in different months are developed and the best fitted theoretical variagrams are determined. For this purpose, different kinds of theatrical variograms including spherical, Gaussian and exponential are considered and the theoretical and empirical variograms have been fitted to each other especially in the region below the sill line. In this procedure different pixel sizes are considered and finally the pixel size of 1° has been selected for further analysis. As an example, the fitted 2D theoretical variogram to the precipitation of May is shown in Figure 2. In Table2 the type, nugget effect, and Range of the 2D and 3D variograms fitted to the precipitation data in each month are presented. The average estimated precipitations for the study region in each month using 2D and 3D Kriging methods based on the selected theoretical varioagrams are shown in Table 3. The results of 2D and 3D Kriging are quite similar to each other. The difference between the results of total annual rainfall is about 1%. The maximum difference is observed in February.

Table 1: The statistics of applying conventional spatial analysis methods in the case study (mm)

Conventional Methods	Moving average			Moving Surface			Nearest Point		
Month	Mean	Median	Std.Dev	Mean	Median	Std.Dev	Mean	Median	Std.Dev
January	70.2	75.9	21. 5	68.8	69.7	27.8	66.0	65.8	31.2
February	56.9	60.7	12.2	57.0	60	19.4	55.2	60.3	23.6
March	67.5	68	9.8	69.4	70.7	19.4	68.3	68.2	25.9
April	47.8	51.1	13.8	49.7	53.4	15.8	50.6	59.6	18.4
May	24.5	20.9	15.1	25.7	25.5	15.7	25.6	28.4	15.9
June	3.4	0.9	4.4	3. 6	1	4.9	3.2	0.8	4.4
July	1.5	1	1.2	1. 6	1.1	1.4	1.5	1	1.4
August	1.0	0.6	0.8	1.1	0.6	1.1	1.9	1.2	2.3
September	1.0	0.3	1.2	1.1	0.3	1.4	1.0	0.6	1.3
October	15.5	15.8	7.1	15.8	17.2	7.5	12.4	11.4	12.7
November	47.0	47.3	9.3	46.5	44.5	14.2	44.9	43.2	15.7
December	70.4	76.4	19.8	69.2	71.9	25.4	67.1	69.7	31.7
Error (%)	16			14			21		

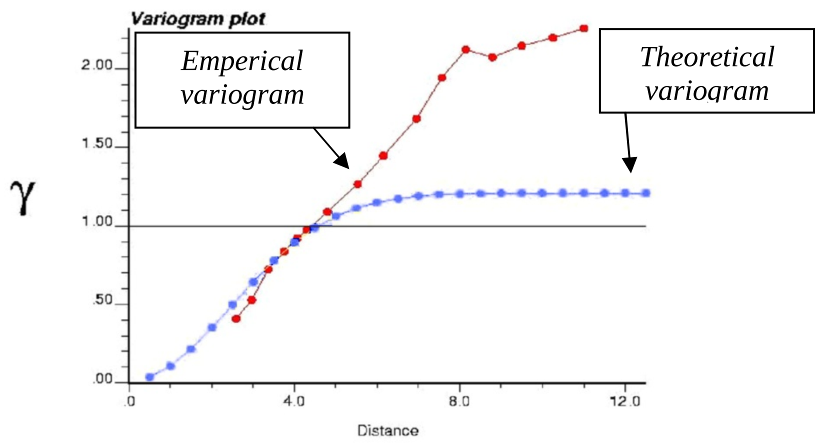


Figure2: Theoretical and Empirical variogram for May

Table2: Type, Nugget effect, and range of variograms for each month (mm)

Kriging Model	2D			3D		
Month	Nugget	h_{max}	Variogram type	Nugget	h_{max}	Variogram type
January	0.8	3	Spherical	0.01	45	Exponential
February	0.85	3	Spherical	0.01	40	Gaussian
March	0.9	5.5	Spherical	0.01	50	Spherical
April	0.4	6.25	Gaussian	0.1	45	Gaussian
May	0.01	6	Gaussian	0.1	50	Gaussian
June	0.01	5.5	Gaussian	0.01	45	Exponential
July	0.1	5.5	Gaussian	1	30	Exponential
August	0.2	5.5	Gaussian	0.01	30	Gaussian
September	0.01	5.5	Gaussian	0.01	30	Spherical
October	0.01	5.8	Gaussian	0	30	Exponential
November	1	5.8	Gaussian	0.01	30	Gaussian
December	0.85	5	Exponential	0.17	30	Exponential

Table 3: Average monthly precipitation estimated using Kriging method (2D and 3D)

Month	Average monthly precipitation (mm) using Kriging 2D	Average monthly precipitation (mm) using Kriging 3D
January	68.48	68.3
February	54.95	59.2
March	67.4	65.8
April	44.44	44.9
May	24.81	22.7
June	3.1	3.2
July	1.51	1.5
August	1.11	1.1
September	1	0.9
October	14.63	14.7
November	45.54	44.7
December	66.8	68.1
Sum	391.07	395

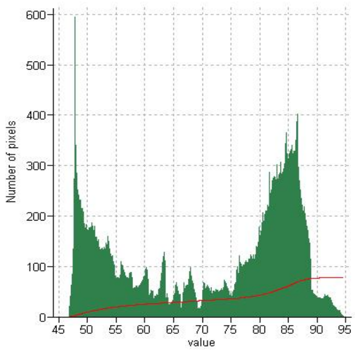
The statistics of the spatial analysis of precipitation in the study region using the advanced Kriging methods (including ordinary, anisotropic, and universal Kriging) are presented in Table 4. The statistics of anisotropic and ordinary Kriging results are too similar to each other. This shows that there is no anisotropy in the rainfall data in the case study so the usage of anisotropic Kriging does not have any advantages to ordinary Kriging. The results of ordinary and anisotropic Kriging are more than all of the conventional methods but the results of universal Kriging are less than all other convenient methods in most months. The results of universal Kriging are more similar to the results from 2D and 3D Kriging methods. High differences are observed in the months of January, February and December when high amounts of rainfall occur.

In the case study, the related histograms and the interpolated maps of rainfall developed by using different methods for January are presented in Figure 3 and Figure 4. The smooth histograms without jumps show better performance of interpolation technique. As can be seen, universal Kriging has the smoothest histogram and there are no considerable jumps in rainfall amount. But the histogram of the nearest point method is not continuous and the amount of precipitation in some adjacent points suddenly changes. There are some jumps in the results of the moving average and moving surface methods. The distribution of the interpolated rainfall in moving surface method is different from other methods to some extent. The results of ordinary and anisotropic Kriging are too similar to each other. This is because the anisotropy angle of the precipitation propagation in this study is too small (less than 5 degrees). The density distribution in universal Kriging is smoother and more legal than the other methods results. The important result in the universal Kriging histogram is the density of points with precipitations which is less than the other methods. It is the only method where jumps in rainfall estimation are not present.

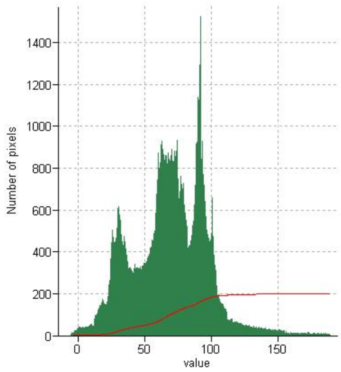
Table 4: The statistics of spatial analysis of rainfall using advanced Kriging methods (mm)

Kriging Methods	**Anisotropic Kriging**			**Ordinary Kriging**			**Universal Kriging**		
Month	**Mean**	**Median**	**Std.Dev**	**Mean**	**Median**	**Std.Dev**	**Mean**	**Median**	**Std.Dev**
January	71.48	77.38	16.04	71.48	77.38	16.04	67.54	73.59	22.82
February	58.02	58.95	7.11	58.02	58.95	7.11	55.21	61.65	15.71
march	66.67	65.81	4.88	66.67	65.81	4.88	65.52	68.92	14.87
April	48.85	53.41	14.18	48.85	53.41	14.18	47.65	47.63	15.81
may	25.31	21.78	15.59	25.31	21.78	15.59	25.28	22.11	15.9
June	3.47	1	4.96	3.47	1	4.96	3.41	0.99	4.97
July	1.47	1.05	1.08	1.47	1.05	1.08	2.24	2.37	3.95
august	1.15	0.72	0.98	1.15	0.72	0.98	1.13	0.72	1.11
September	1.03	0.31	1.37	1.03	0.31	1.37	1.01	0.31	1.39
October	16.03	17.28	7.82	16.03	17.28	7.82	16.02	17.19	7.84
November	47.55	51.5	7.45	44.41	45.63	12.03	44.41	45.63	12.03
December	70.61	77.72	12.88	70.61	77.72	12.88	65.76	72.61	20.4

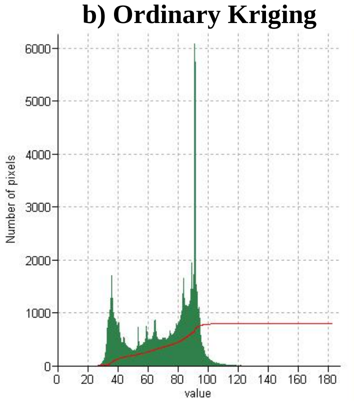
As Kriging errors can be evaluated in different points of the study area, confidence interval maps are developed for evaluation of the certainty of the interpolated results. The confidence interval map is developed by combining the outputs and related errors of the Kriging method. When the errors have a normal distribution, the critical values of confidence levels could be found in a probability distribution table of a standard normal curve. For this purpose the $\mu \pm c\sigma$ bounds are determined in which μ is the reference level and σ is the estimated error (standard deviation). The multiplication factors c (critical value) for the estimated errors (σ) in the error map for different one-sided confidence levels are presented in Table 5.



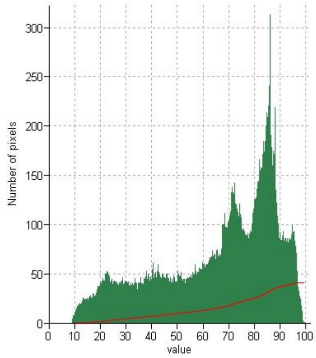
a) Anisotropic Kriging



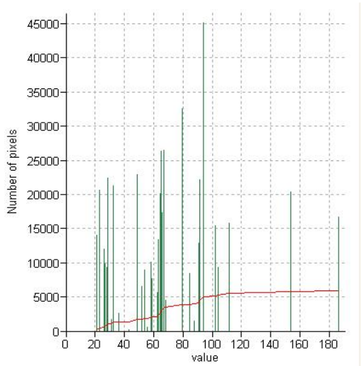
b) Ordinary Kriging



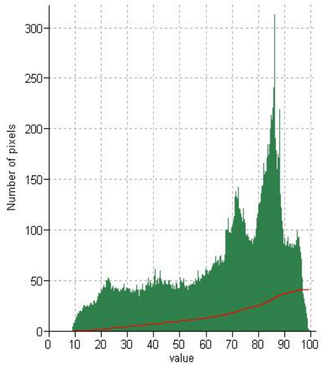
c) Moving Average



d) Moving Surface



e) Nearest Point



f) Universal Kriging

Figure 3: The related histograms for January developed using different methods

a) Anisotropic Kriging

b) Ordinary Kriging

c) Moving Average

d) Moving Surface

e) Nearest Point

f) Universal Kriging

Figure4: The interpolated maps of rainfall for January developed using different methods

Table 5: The multiplication factors for different confidence levels

Confidence level:	90%	95%	97.5%	99%	99.5%
Critical value *c*:	1.282	1.645	1.960	2.326	2.576

The kriging estimations which fall between these criteria are considered as the uncertain estimations in this study. Different levels of certainty have been considered and the highest level has been reported as the certainty level.

The results of the universal and ordinary Kriging are evaluated in the same way and the results are presented in Table 6. As can be seen in this table, months with less precipitation, have a lower level of confidence. This might be because of the high impact of the local events that can influence low amounts of precipitation and increase the uncertainty in these months.

5-Optimal design of rain gauges network

In this study, cross validation is used for error evaluation and for optimal design of rain gauge network. For this purpose, using the selected models and the kriging equations, the $Z(x_i)$ variable is measured by omitting $Z(x_i)$ from the input data. The value of $\hat{Z}(x_i)$ can then be estimated. Theoretical and observed variograms have been compared and the interpolation errors are determined as follows:

$$\text{Error}=\left|\frac{Z(x_i)-\hat{Z}(x_i)}{Z(x_i)}\right| \quad (9)$$

Table 6: The certainty analysis of the Kriging outputs (mm)

Month	Universal Kriging		Ordinary Kriging	
	Percent of uncertain area	Level of confidence	Percent of uncertain area	Level of confidence
January	7.04	99.5	5.19	99.5
February	8.52	99.5	0	99.5
March	12	90	7.8	90
April	7.04	99.5	2.59	99.5
May	0.74	99.5	0.74	99.5
June	9.6	95	9.63	99.5
July	9.26	90	7	90
August	5.93	95	5.93	95
September	8.89	95	7.04	95
October	1.11	99.5	1.11	99.5
November	7	90	5.9	90
December	8.15	99.5	8.89	99.5

The GSlib software used in this study computes errors and error variance which accordingly are considered for placement of new rain gauges. Monthly maps for errors and variance of errors have been developed (total of 24 maps). For considering the combined monthly effects, the maps of total variations of errors (Figure 6) and total variance of errors (Figure 6) are developed. Figure 6 shows that the range of variation of variance of errors is very limited (only 0.35), therefore it can not be used for selection of new rain gauge locations. But as it can be seen in Figure 5, the variation of errors in different pixels of the study area is high. So this map has been used to determine the new gauge locations. The larger values of errors are occurred in the southern part of the study area. Pixels with errors of more than 4.5 (about 75% of maximum error) have been considered for the new stations and the map of summation of errors has been filtered for these pixels (Figure 7). Considering distribution of filtered pixels, seven locations have been suggested for new stations as shown in Table 7.

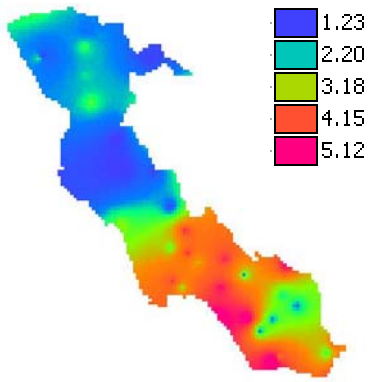


Figure 5: Map of regional estimation errors

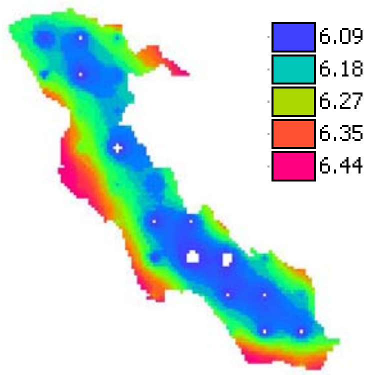


Figure 6: Map of regional estimation variance of errors

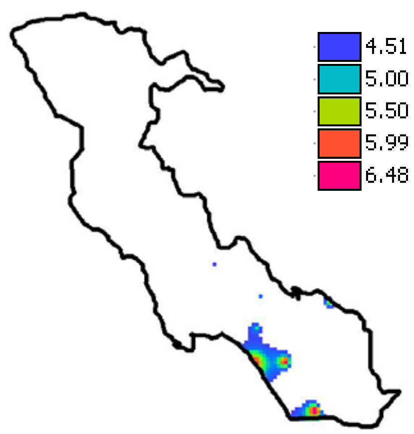


Figure 7: Filtered pixels with errors more than 4.5

Table 7-Location and number of proposed gauges

No. of gauge	Latitude Tolerance		Longitude Tolerance	
1	48° 42'	48° 47'	32° 47'	32° 52'
2	49° 47'	49° 52'	32° 02'	32° 07'
3	51° 22'	51° 27'	31° 52'	31° 57′
4	49° 42'	49° 47'	31° 17′	31° 22′
5	49° 37'	49° 42'	30° 27'	30° 32'
6	50° 22	50° 27'	30° 27'	30° 32'
7	51° 02'	51° 07'	29° 22′	29° 27′

6-Conclusion

In this study the spatial analysis of precipitation is investigated using different convenient interpolation techniques and Kriging methods for spatial analysis of precipitation in the western region of Iran. The results of different methods are compared based on the monthly interpolated precipitation statistics, the resulting interpolated maps, and the developed histograms. Depending on the basic characteristics and assumptions of different methods there were some meaningful differences in the results. Universal Kriging showed a more stable and smooth behavior among different methods. There were some differences in its histogram shape but it was still the most legal histogram because of its continuity and lack of jumps. Five points in the study area are considered as benchmarks for comparison of the results. Based on these points, universal Kriging had the least error equal to 4% where other techniques had errors of more than 10%. This showed that local effects and relations must be considered in spatial analysis of precipitation. Cross validation was used for error evaluation and for placement of new rain gauges. Finally, seven new stations have been proposed.

7-Acknowledgments

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