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Technical Report

گزارش فنی

Parameter Estimation of Nash Conceptual Model Using Genetic Algorithm and Ordinary Least Square Methods

تخمین پارامترهای مدل مفهومی ناش با استفاده از روش‌های الگوریتم ژنتیک و حداقل مربعات معمولی

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Abstract

This study is focused on two parameter estimation methods of Ordinary Least Square (OLS) and binary code Genetic Algorithm (GA) for estimating the *k*, *n* Nash conceptual model parameters. The efficiency of these methods is compared by applying the calibrated models in simulating seven rainfall-runoff events in Heng-Chi watershed in north of Taiwan. The results of the goodness of fit criteria indicate that GA method has better performance in terms of coefficient of efficiency and has reduced the coefficient of variation of error in simulated discharge and error in peak discharge.

چکیده

در این تحقیق، دو روش حداقل مربعات معمولی و الگوریتم ژنتیک دو- دویی به منظور تخمین پارامترهای مدل مفهومی ناش (n, k) مورد استفاده قرار گرفته است. کارآیی این دو روش با بکارگیری پارامترهای تخمینی در شبیه سازی هفت واقعه بارندگی-رواناب واقع در حوضه Heng-Chi در شمال تایوان مورد ارزیابی قرار گرفت. نتایج حاصل از معیارهای نکوئی برازش در مورد نتایج هر دو مدل نشان داد که مدل الگوریتم ژنتیک قادر به بهبود معیار ضریب کارآئی و کاهش ضریب تغییرات و خطای دبی اوج مدل نسبت به روش حداقل مربعات می باشد.

Keywords: Parameter estimation, Nash conceptual model, Ordinary Least Square, Genetic Algorithm.

کلمات کلیدی: تخمین پارامتر، مدل مفهومی ناش، حداقل مربعات معمولی، الگوریتم ژنتیک.

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Introduction

Lumped conceptual types of models (LCM) such as the Nash conceptual model, as shown in Fig. 1, usually have few parameters which cannot be determined directly from physical catchment characteristics and therefore should be estimated by parameter estimation methods. Mathematical parameter estimation methods such as Ordinary Least Square (OLS) rely on mathematical properties of the error response surface. In OLS method, one often encounters a situation in which the derived Unit Hydrograph (UH) exhibit noise fluctuation among its ordinates. This is mainly caused by nonlinearity, time variance, distributed space, sampling intervals for time, and measurement error (Dooge et al., 1989). In recent years, evolutionary methods such as the Genetic Algorithm (GA) have been successfully used in the field of hydrology and water resources to directly map nonlinear complex relations. Researches indicate that GA may be an efficient and robust means of calibrating for a variety of models where measured time-series data are available (Wang, 1991; Mohan and Loucks, 1995).

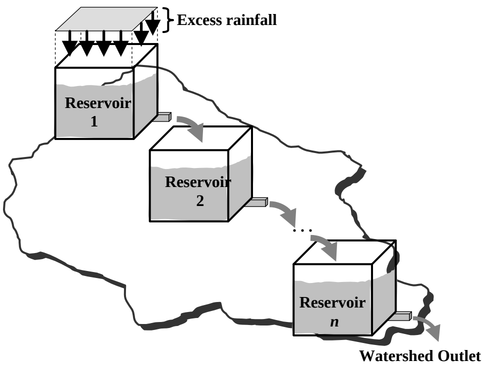

Figure 1- Schematic of cascade of reservoirs in Nash conceptual model

Nash Conceptual Model

The Nash conceptual model as a two-parameter gamma distribution has the following structure:

$$u(t) = \frac{1}{k\Gamma(n)}\left(\frac{t}{k}\right)^{(n-1)} e^{-t/k} \qquad (1)$$

$u(t)$: ordinate of Instantaneous Unit Hydrograph (IUH) at time t,
t: time since commence of rainfall and runoff,
*n:*number of linear reservoirs of equal storage coefficient,
k: storage coefficient of reservoirs (Nash, 1957).
The merit of this model for UH derivation is that it provides all possible shapes depending on the magnitude of its parameters.

Parameter Estimation Methods

In the GA model, the Root Mean Square of Error (RMSE) of the simulated values obtained from the Nash model is considered as the fitness function. The GA model, therefore, searches for the optimal solution which minimizes the simulation errors. MATLAB software has been used for formulating and running the GA model.

In OLS method, Nash model is linearized by applying the logarithm operator as follows:

$$\ln Q = -\ln k^n \Gamma(n) - \frac{t}{k} + (n-1)\ln t \qquad (2)$$

With replacing the Y=ln$Q$, X=ln$t$, C=n-1, B=-1/k, $A = -\ln k^n \Gamma(n)$, and Z=t, equation 2 will have a linear form as follows:

$$Y=A+BZ+CX \qquad (3)$$

Where B and C must be estimated by OLS method.

To evaluate the goodness of fit for the models, three criteria are selected (Khu, 1998):
1. Nash-Sutcliffe coefficient (R^2):

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left[Q_{obs,t} - Q_{sim,t}\right]^2}{\sum_{i=1}^{N}\left[Q_{obs,t} - \overline{Q}_{obs}\right]^2} \qquad (4)$$

2. Coefficient of Variation (CV):

$$CV = \frac{1}{\overline{Q}}\left(\frac{\sum_{i=1}^{N}\left[Q_{obs,t} - Q_{sim,t}\right]^2}{N}\right)^{1/2} \qquad (5)$$

3. Error in Peak Discharge (EPD):

$$EP = \frac{Q_{PEAK,sim} - Q_{PEAK,obs}}{Q_{PEAK,obs}} \qquad (6)$$

where $Q_{obs,i}$= observed flow in time t (m^3/s); $Q_{sim,i}$= simulated flow in time t (m^3/s); $\overline{Q}_{obs}$ = average flow in each event (m^3/s); N = number of time steps in an event; $Q_{peak,sim}$= simulated peak flow (m^3/s); and $Q_{peak,obs}$= observed peak flow (m^3/s).

Case Study

Seven rainfall-runoff events in Heng-Chi watershed (Fig. 2) which is located in northern Taiwan was selected to investigate the applicability of the two parameter estimation approaches. The characteristic of these events are shown in Table 1. In this study, constant slope method has been used for base flow separation and direct runoff data has been used for conducting the analysis.

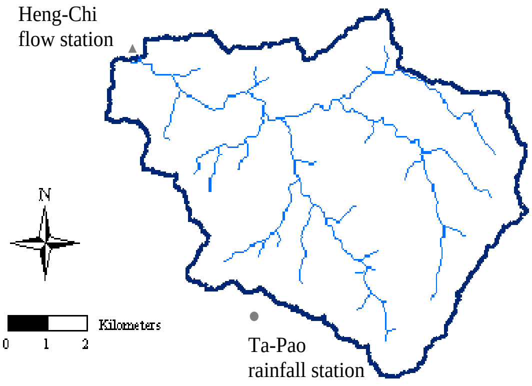

Figure 2- The Heng-Chi watershed and Location of gauging stations

Table 1- Characteristics of rainfall-runoff events

Date	Direct runoff			Excess rain	
	Peak (m^3/s)	Time to Peak (hr)	Duration (hr)	Depth (mm)	Duration (hr)
10/31/2000	308.21	18	59	329.53	27
07/30/1996	199.00	30	40	133.00	36
07/10/1994	49.90	12	46	27.95	10
06/05/1993	156.94	11	20	72.20	13
08/18/1990	468.00	32	61	235.20	39
07/27/1987	140.00	7	33	54.35	9
09/17/1984	158.40	67	37	246.00	49

Results and Discussion

Four rainfall-runoff events shown in Table 1 were selected for calibration and parameter estimation and three other events were used for model verification. The optimum values of cross-over and mutation probability, initial population, and the number of generations for the GA model are estimated as 0.7, 0.014, 40, and 150, respectively. The estimated values of k and n for the four rainfall-runoff events using the GA model are 5.43 and 4.46, and for the OLS method are 5.53 and 4.37, respectively. The remaining three events were also simulated to check the validity of the models. For instance Fig. 3 shows two of the simulated events. As it can be seen in this figure, both models have been successfully able to simulate the overall shape and the amount of peak and time-to-peak of the hydrographs.

The three criteria shown in equations 4, 5, and 6 were applied to all three events which considered for validation and the results are presented in Table 2.

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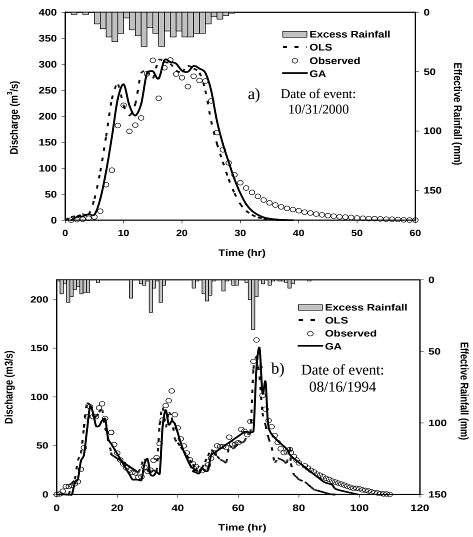

Figure 3- Comparison between simulated and observed hydrographs of the events used for model validation

Table 2- Results of goodness of fit of the models

Event	R^2		EP		CV	
	OLS	GA	OLS	GA	OLS	GA
10/31/2000	0.75	0.79	-4.31	-3.80	0.63	0.58
07/30/1996	0.93	0.96	-0.76	-0.23	0.29	0.23
07/10/1994	0.74	0.79	-3.06	-2.85	0.52	0.46

The values in Table 2 indicate that the simulated events using the GA method have the higher coefficient of efficiency and lower variation and error in peak discharge compared to the OLS method.

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